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The influence of nonlinear magnetic pull on hydropower generator rotors

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Abstract

In large electrical machines the electromagnetic forces can in some situations have a strong influence on the rotor dynamics. One such case is when the rotor is eccentrically displaced in the generator bore. A strong unbalanced magnetic pull will then appear in the direction of the smallest air-gap. In this paper, the influence of nonlinear magnetic pull is studied for a hydropower generator where the generator spider hub does not coincide with the centre of the generator rim.

The generator model consists of a four-degree-of-freedom rigid body, which is connected to an elastic shaft supported by isotropic bearings. The influence of magnetic pull is calculated for the case when the generator spider hub deviates from the centre of the generator rim. A nonlinear model of the magnetic pull is introduced to the model by radial forces and transverse moments.

In the numerical analysis input parameters typical for a 70 MW hydropower generator are used. Results are presented in stability and response diagrams. The results show that this type of rotor configuration can in some cases become unstable. Therefore, it is important to consider the distance between the centreline of generator spider hub and the centreline of generator rim.

1. Introduction

The occurrence and the effect of an eccentric rotor in an electric machine have been discussed for more than one hundred years. During operation, the rotor eccentricity gives rise to a non-uniform air-gap which produces an unbalanced magnetic pull acting on the rotor. The influence from the unbalanced magnetic pull on the rotating system can be severe since instability of the shaft can occur. There are a number of documented cases where the rotor has been in contact with the stator due to air-gap asymmetry [1].

Knowledge of the radial magnetic forces acting on the rotor in an eccentric machine is important for the mechanical design of the rotor. A number of equations have been suggested for the calculation of the magnetic pull on a rotor due to a disturbance in the magnetic field. Early papers such as Behrend [2], Gray [3] and

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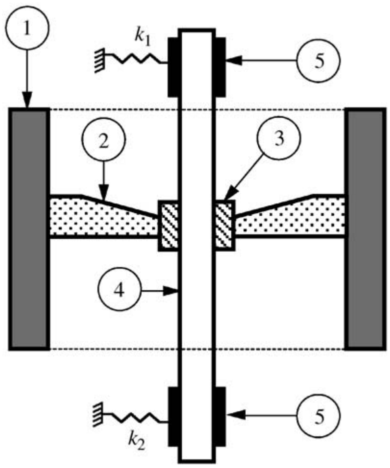

Fig. 1. Schematic figure of the generator rotor model consisting of (1) generator rim including the poles, (2) rotor spider, (3) rotor spider hub connected to the (4) shaft, (5) the generator rotor is radial supported in the two bearings.

Robinson [4] suggested linear equations for calculation of the magnetic pull for an eccentricity up to 10% of the average air-gap. Covo [5] and Ohishi et al. [6] improved the equations for calculation of magnetic pull by taking into account the effect of saturation on the magnetization curve. A more general theory was suggested by Belmans et al. [7,8] where they developed an analytic model for vibrations in induction motors. They showed that the unbalanced magnetic pull acting on the rotor also consists of harmonic components. This harmonic component has a double slip frequency if the rotor is dynamically eccentric or double supply frequency for a statically eccentric machine. Investigation of radial stability of a flexible shaft in two pole induction machines was performed by Belmans et al. [9] to verify the theoretical results. They studied the influence of magnetic pull on the rotor system for various values of the slip and supply frequency. A similar study was performed by Guo et al. [10]. They studied the effects of unbalanced magnetic pull and the vibration level in three-phase generators with any number of pole pairs.

The usual way to calculate the influence of magnetic pull on a rotor is to apply the radial pull force at the generator spider hub, Fig. 1 gustavsson et al. [11] suggested a linear model where the distance between the spider hub and the generator rim centre is taken into consideration. In this paper, the influence of nonlinear magnetic pull on the rotor stability is studied for a synchronous hydropower generator where the generator spider hub does not coincide with the axial centre of the generator rim. An analytical model of the bending moments and radial forces acting at the generator hub is developed where the distance between the generator spider hub and the generator rim centre is taken into consideration. A simple generator model is then combined with the developed load model. Using this simple rotor model, numerical results are presented to describe the effects of unbalanced magnetic pull acting on a displaced generator rim.

2. Unbalanced magnetic pull on a generator rotor

The air-gap flux in an electrical machine gives rise to large radial attraction forces between the stator and the rotor. In a perfectly symmetric machine, the sum of all these radial magnetic forces acting on the rotor is equal to zero, which means that there is no radial net force acting on the rotor. However, all hydropower generators are associated with some degree of asymmetry in the air-gap [12].

The most common example of such asymmetry is the so-called air-gap eccentricity. Air-gap eccentricity occurs if the centre of the rotor does not coincide perfectly with the centre of the stator bore. The relative eccentricity, e', is defined as

$$e' = \frac{u_r}{R_s - R_r} = \frac{u_r}{\Delta r}, \tag{1}$$

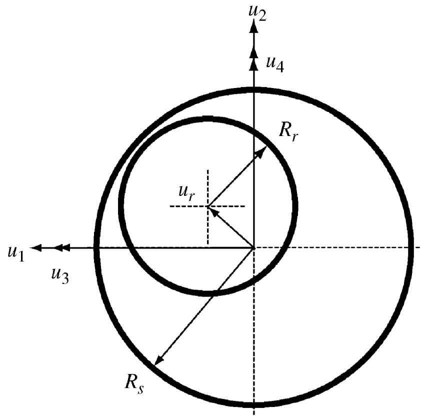

Fig. 2. Air-gap of a generator with an eccentric rotor.

where u_r is the distance between the centre of the rotor and the centre of the stator, R_s is the inner radius of the stator, R_r is the outer radius of the rotor and Δr is the average air-gap between the rotor and stator. See Fig. 2.

An analytical expression for the unbalanced magnetic pull caused by static and dynamic eccentricity was presented by Belmans et al. [8] and Sandarangani [12] for a three-phase electrical machine with an arbitrary number of poles. The time-dependent attractive force is composed of a constant force and an alternating force. The alternating force oscillates with twice the supply frequency for static eccentricity, and with twice the supply frequency multiplied by the slip for dynamic eccentricity. Sandarangani [12] showed that the alternating force decreases with an increasing number of poles in the machine. For a hydropower generator with many poles the alternating magnetic pull force can be neglected due to its insignificance in comparison to the constant magnetic pull force.

The expression for the value of the constant unbalanced magnetic pull force, f_e, for a rotor parallel to the stator was found from the integration of the horizontal and vertical projection of the Maxwell stress over the rotor surface.

$$f_e = \frac{\mu_0}{2}\frac{S_s^2}{p^2}\frac{R_s^3 h\pi}{\Delta r^2}\frac{e'}{\sqrt{\left(1-e'^2\right)^3}}, \tag{2}$$

where S_s is the stator linear current density, p is the number of pole pairs, h is the length of the rotor and μ_0 is the permeability of free space. Furthermore, the direction of the constant unbalanced magnetic pull is always towards the smallest air-gap between the stator and the rotor.

In a generator, the most common case of eccentricity is a combination of stator eccentricity and rotor eccentricity. The characteristic for stator eccentricity is that the rotor centre will be in a fixed position in the stator bore under the action of a constant magnetic force. Due to this, the smallest air-gap vector will always appear in the same direction relative to the stator. In the case of rotor eccentricity, the rotor centre will whirl around the rotor centreline. If a stator eccentricity is combined with rotor eccentricity the rotor centre will whirl around the static eccentricity point and hence the smallest air-gap will fluctuate with the whirling frequency.

3. The model of the generator rotor

The rotor model consists of a massless uniform vertical shaft of length L, supported in a tilting-pad bearing at each end with stiffnesses k_1 and k_2, respectively, as shown in Fig. 3. The shaft has second moment of area I, and Young's modulus E. The generator that is connected to the shaft is treated as a rigid body, consisting of a generator spider, generator rim and poles. The generator is considered to have total mass m, polar moment of inertia J_p, transverse moment of inertia J_t and length h.

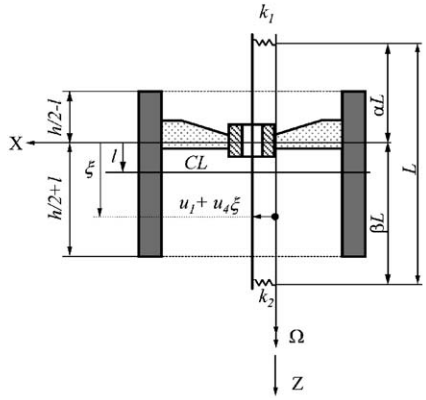

Fig. 3. Generator rotor displaced a distance u_1+$u_4\xi$ from the generator vertical line.

The rotor geometry and dimension parameters are shown in Fig. 3. The distance between the axial centre of the generator rim and the centre of the generator spider hub is denoted by l. The position of the generator spider hub along the shaft is denoted by αL where $\alpha+\beta=1$. Let u_1 be a positive displacement in X-direction, u_4 the rotation around the Y coordinate axis and ξ the distance from the centre of the rotor hub to a cross section on the rotor rim. Then the displacement (small angles) of a specific section of the generator rotor in the X-direction, is given by u_1+$u_4\xi$. In the same way, if u_2 is positive displacement in the Y-direction and u_3 is the rotation around the X coordinate axis, the displacement (small angles) of the generator rotor at a distance ξ from the rotor hub in Y-direction is u_2−u_3 ξ. The angular velocity of the generator rotor about the Z-axis is denoted Ω. The connection of the generator rotor hub to the shaft is assumed to be stiff.

4. Rotor unaffected by magnetic force

The displacement vector $\mathbf{u}=\{u_1\ u_2\ u_3\ u_4\}^{\mathrm{T}}$ of the shaft, which is simply supported in the bearings, depends on the force vector $\mathbf{f(t)}$ acting on the shaft at the position of the centre of the rotor hub. The displacement and the curvature of the shaft can be derived from the integration of the bending torque distribution in the shaft. To obtain the total displacement and rotation of the generator rotor (at the position of the rotor hub) the deflections in the bearings have been added to the shaft displacement. From the simple model of the rotating system the flexibility $\mathbf{u}=\mathbf{Af}(t)$ with the parameters shown in Fig. 3, and under the assumption that the bearings and rotor are isotropic, the flexibility matrix $\mathbf{A}$ can be formulated as

$$\mathbf{A}=\begin{bmatrix} A & 0 & 0 & -B \\ 0 & A & B & 0 \\ 0 & B & C & 0 \\ -B & 0 & 0 & C \end{bmatrix}, \tag{3}$$

where

$$A=\frac{L^3\alpha^2\beta^2}{3EI}+\frac{\alpha^2}{k_2}+\frac{\beta^2}{k_1},$$
$$B=\frac{L^2\alpha\beta(\alpha-\beta)}{3EI}-\frac{1}{L}\left(\frac{\alpha}{k_2}-\frac{\beta}{k_1}\right),$$
$$C=\frac{L(1-3\alpha\beta)}{3EI}+\frac{1}{L^2}\left(\frac{1}{k_2}+\frac{1}{k_1}\right).$$

The rotor stiffness matrix $\mathbf{K}$ can be formaulated as the inverse of the flexibility matrix, $\mathbf{K} = \mathbf{A}^{-1}$:

$$\mathbf{K} = \frac{1}{AC - B^2}\begin{bmatrix} C & 0 & 0 & B \\ 0 & C & -B & 0 \\ 0 & -B & A & 0 \\ B & 0 & 0 & A \end{bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 & k_{12} \\ 0 & k_{11} & -k_{12} & 0 \\ 0 & -k_{12} & k_{22} & 0 \\ k_{12} & 0 & 0 & k_{22} \end{bmatrix}. \tag{4}$$

The equation of motion for the generator rotor model shown in Fig. 3 can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{u}} + (\Omega\mathbf{G} + \mathbf{C})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t), \tag{5}$$

where $\mathbf{M}$ is a diagonal mass matrix, including transverse moment of inertia of the generator rotor, $\mathbf{G}$ is the skew symmetric gyroscopic matrix, $\mathbf{K}$ is the stiffness matrix of the shaft, $\mathbf{C}$ is the damping matrix for the rotor and Ω the rotational speed of the rotor. In this paper a damping matrix proportional to the shaft stiffness $\mathbf{C} = \gamma\mathbf{K}$ is assumed. The term $\mathbf{f}(t)$ is a time-dependent load vector.

5. Rotor affected by magnetic force

Extension of the rotor model described in Eq. (5) to include the nonlinear magnetic rotor pull as well as a generator spider hub which is not coincident with the centre of the generator rim, $l \neq 0$, will be presented.

The generator eccentricity causes a disturbance in the magnetic field, which results in a pull force and torque acting on the generator spider hub. The magnetic force f_e depends on the rotor displacement $u_1(x-\text{displ.})$, $u_2(y-\text{displ.})$, inclination of the rotor $u_3(x-\text{rot.})$, $u_4(y-\text{rot.})$ and the distance l between generator spider hub and the geometrical centre of the generator rim. The inclination of the rotor is assumed to be small which gives the cosine of inclination angle to be approximately equal to 1 and the sine of the angle to be the angle itself.

The magnetic pull force acting on a unit length of the rotor rim has been obtained by dividing the magnetic pull force in Eq. (2) by the length h of the rotor rim. By assembling the constants in Eq. (2) into term k_e, Eq. (2) can be rewritten for a load element as

$$\mathrm{d}f_e = \frac{k_e u_r(\xi)}{h\sqrt{\left(1 - \left(u_r(\xi)/\Delta r\right)^2\right)^3}}\,\mathrm{d}\xi, \tag{6}$$

where the $\mathrm{d}\xi$ is the length of the element in the axial direction. The magnetic pull force for a rotor not parallel to the stator can be obtained by substituting the rotor displacements $u_r(\xi)$ in Eq. (6) with a function for the rotor displacement at a specific distance (ξ) from the rotor hub. The magnetic pull force in the X-direction can be found by substituting the rotor displacement $u_r(\xi)$ in Eq. (6) with $u_{r_1}(\xi) = u_1 + u_4\xi$, and in the Y-direction the substitution is $u_{r_2}(\xi) = u_2 - u_3\xi$. The integration of Eq. (6) over the rotor height h from the centre of the rotor hub gives

$$\left.\begin{aligned} f_{e_i} &= \frac{k_e}{h}\int_{-(h/2)+l}^{(h/2)+l} \frac{u_{r_i}}{\sqrt{\left(1 - \left(u_{r_i}/\Delta r\right)^2\right)^3}}\,\mathrm{d}\xi \\ &\approx \frac{k_e}{h}\int_{-(h/2)+l}^{(h/2)+l} u_{r_i}\left(1 + \frac{3}{2}\left(\frac{u_{r_i}}{\Delta r}\right)^2 + \frac{15}{8}\left(\frac{u_{r_i}}{\Delta r}\right)^4 + \frac{35}{16}\left(\frac{u_{r_i}}{\Delta r}\right)^6 + \frac{315}{64}\left(\frac{u_{r_i}}{\Delta r}\right)^8\right)\mathrm{d}\xi \end{aligned}\right\} \quad \text{for } i = 1, 2. \tag{7}$$

The torque acting on the generator spider hub depends on the vertical position l of the rotor rim, the rotor displacements u_1, u_2 and the inclinations u_3, u_4 of the rotor. The torque o1n the rotor hub due to the magnetic pull force (as acting on a rotor element $\mathrm{d}\xi$ at a distance ξ from the rotor hub) can be found by calculating the torque from the magnetic pull force acting on the rotor rim. The torque around the X coordinate axis can be formulated by substituting the displacements $u_r(\xi)$ in Eq. (6) with $u_{r_3}(\xi) = -u_2 + u_3\xi$ and the torque around the Y coordinate axis can be formulated by the substitution of $u_{r_4}(\xi) = u_1 + u_4\xi$. Multiplication of Eq. (6) with

ξ and integration over the rotor height h from the centre of the rotor hub then gives

$$\left.\begin{aligned} f_{e_i} &= \frac{k_e}{h}\int_{-(h/2)+l}^{(h/2)+l} \frac{u_{r_i}\xi}{\sqrt{\left(1-\left(u_{r_i}/\Delta r\right)^2\right)^3}}\mathrm{d}\xi \\ &\approx \frac{k_e}{h}\int_{-(h/2)+l}^{(h/2)+l} \xi u_{r_i}\left(1+\frac{3}{2}\left(\frac{u_{r_i}}{\Delta r}\right)^2+\frac{15}{8}\left(\frac{u_{r_i}}{\Delta r}\right)^4+\frac{35}{16}\left(\frac{u_{r_i}}{\Delta r}\right)^6+\frac{315}{64}\left(\frac{u_{r_i}}{\Delta r}\right)^8\right)\mathrm{d}\xi \end{aligned}\right\} \quad \text{for } i=3,4. \tag{8}$$

The stator eccentricity results in a constant magnetic pull force and torque acting on the rotor. This load vector can be calculated according to Eqs. (7) and (8) for a specific stator eccentricity. The equation of motion for the generator rotor model shown in Fig. 3, including the effects from the magnetic pull and the displacement of the generator rim from the spider hinge line, can be written in matrix form as

$$\mathbf{M}\ddot{\mathbf{u}} + (\Omega\mathbf{G}+\mathbf{C})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) + \mathbf{f_e}(\mathbf{u}), \tag{9}$$

where $\mathbf{f}(t)$ is a time-dependent load vector and $\mathbf{f_e}(\mathbf{u})$ is the magnetic pull force acting on the rotor due to the rotor displacement $\mathbf{u}$.

In hydropower applications it is not unusual that the generator stator centreline does not coincide with the centreline of the rotor. The reason for the misalignment can for instance be a non-uniform thermal expansion or non-circular stator shape. In this paper this deviation (without magnetic pull and zero rotational speed) has a fixed direction and is denoted as initial eccentricity. If this initial eccentricity is denoted $\mathbf{u_{ie}}$, the equation of motion (9) becomes

$$\mathbf{M}\ddot{\mathbf{u}} + (\Omega\mathbf{G}+\mathbf{C})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) + \mathbf{f_e}(\mathbf{u}+\mathbf{u_{ie}}). \tag{10}$$

6. Stability of stationary points

The stability of the second-order differential equation of motion can be found by rewriting Eq. (10) as a system of first-order differential equations. By defining a state vector $\mathbf{U} = \begin{Bmatrix}\mathbf{u}\\ \dot{\mathbf{u}}\end{Bmatrix}$, Eq. (10) can be written in state vector form as

$$\dot{\mathbf{U}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M^{-1}K} & -\mathbf{M}^{-1}(\Omega\mathbf{G}+\mathbf{C}) \end{bmatrix}\mathbf{U} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{M^{-1}f_e}(\mathbf{u}+\mathbf{u_{ie}}) + \mathbf{M^{-1}f}(t) \end{Bmatrix}. \tag{11}$$

The stationary point $\mathbf{U_s}$ can be found by assuming that $\mathbf{f}(t)$ is equal to zero and setting the time derivatives of Eq. (11) equal to zero. This stationary point is the rotor position at operating conditions. By defining the components of a matrix $D\mathbf{f_e}$ as

$$Dfe_{ij} = \frac{\partial f_e(u+u_{ie})_i}{\partial u_j}, \tag{12}$$

the Jacobian matrix $\mathbf{J}$ evaluated at the stationary point $\mathbf{U_s}$ can be expressed in standard form as

$$\mathbf{J} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}(\mathbf{K}-D\mathbf{f_e}) & -\mathbf{M}^{-1}(\Omega\mathbf{G}+\mathbf{C}) \end{bmatrix}, \tag{13}$$

where the components of the matrix $D\mathbf{f_e}$ are defined as the magnetic stiffness and appear as a negative stiffness matrix which affects the rotor stiffness matrix. Assuming a solution on the form $\mathbf{U} = \mathbf{U_0}\mathrm{e}^{\lambda t}$ the stability of the equation of motion (11) can be studied by the eigenvalue problem

$$(\mathbf{J}-\lambda\mathbf{I})\mathbf{U_0} = \mathbf{0}, \tag{14}$$

where $\mathbf{I}$ is the identity matrix. The solutions consist of eight complex eigenvalues λ and associated eigenvectors $\mathbf{U_0}$. These eigenvalues can be expressed in the general form

$$\lambda = \tau + \mathrm{i}\omega, \tag{15}$$

where τ is the decay rate, and ω is the damped natural frequency of each eigenvalue. The system is asymptotically stable if all eigenvalues have negative decay rates.

7. Analysis of rotor response

The rotor response which depends on the magnetic pull force and the rotor eccentricity can be found by numerical simulation of Eq. (10). The displacement vector $\mathbf{U_{ie}}$ in Eq. (10) represents the initial stator eccentricity. The stator eccentricity results in a constant magnetic pull force acting on the rotor.

Eq. (11) is a system of first-order differential equations which can be solved by ordinary mathematical programs. To solve this system of equations MATLAB's ODE solver has been used which is based on the Runge–Kutta–Fehlberg method.

8. Numerical results

8.1. Rotor stability

The suggested method to include the magnetic pull in the rotor analysis formulated in the previous section has been applied to the rotor model shown in Fig. 3. In the analysis, shaft dimensions and values of magnetic pull have been selected to model a 70 MW three-phase synchronous hydropower generator.

The influence of nonlinear magnetic pull on the generator rim where the generator spider hub and the centre of the generator rim do not coincide has been studied. In Table 1 the numerical data used in the analysis are presented.

The stability of a hydropower generator has been determined for the damped system according to Eq. (14). The stability of the rotor system has been studied with the assumption of proportional damping $\mathbf{C} = \gamma\mathbf{K}$ with $\gamma = 0.003$. In Figs. 3–6 the stability for different rotor configurations has been analysed according to Eq. (15). The generator spider hub position α on the shaft has been analysed for $\alpha = 0$ to 1 and the generator-offset l/h has been varied from -0.5 to 0.5. The selected values of α and l/h cover all possible combination of spider hub positions and generator rotor offset. In Figs. 4–7 the rotor configurations with negative decay rate τ have been marked with black dots. The rotor stability has been analysed for specific values of the stator displacement $\mathbf{u_{ie}}$. In these figures the stator displacement vector $\mathbf{U_{ie}}$ has been defined as $\{u_{\text{stator}}\ 0\ 0\ 0\}^{\text{T}}$ and the value of u_{stator} has been varied between 1% and 7% of the nominal air-gap between the rotor and the stator in the generator. A normally accepted value of the stator eccentricity u_{stator} for a new generator is 3–4% of the air-gap, however for old generators the eccentricity can exceed this value due to factors such as thermal expansion of the stator and may appear in any direction in the X–Y plane.

Table 1
Dimensions and parameters used in the calculations

	Item	Value
E	Young's modulus (N/m^2)	2.0×10^{11}
m	Mass of rotor (kg)	336 000
J_p	Polar moment of inertia (kg m^2)	5.3×10^6
J_t	Transverse moment of inertia (kg m^2)	3.3×10^6
L	Length of shaft (m)	6.5
h	Length of rotor (m)	3
k_e	Magnetic pull constant (N/m)	329.3×10^6
I	Moment of inertia (m^4)	0.0201
k_1	Bearing stiffness (N/m)	7×10^8
k_2	Bearing stiffness (N/m)	7×10^8

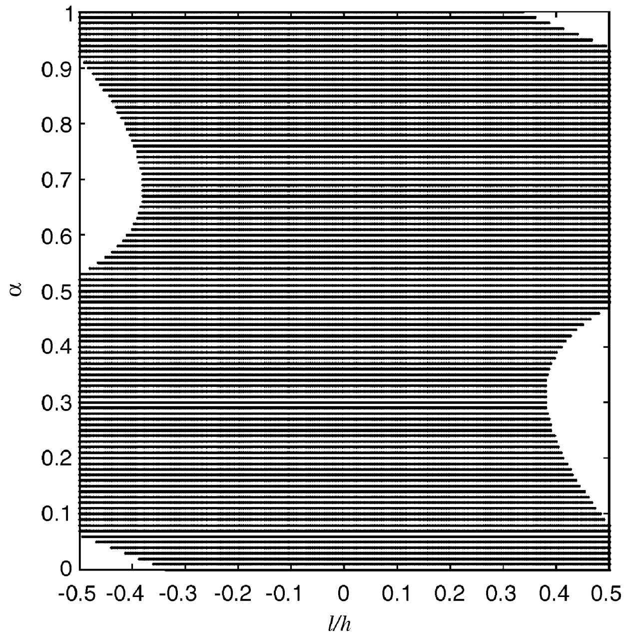

Fig. 4. Stable region (black) for stator displacement u_{Stator} of 1% of the air-gap. Stable region as function of generator position α and generator rotor offset l/h.

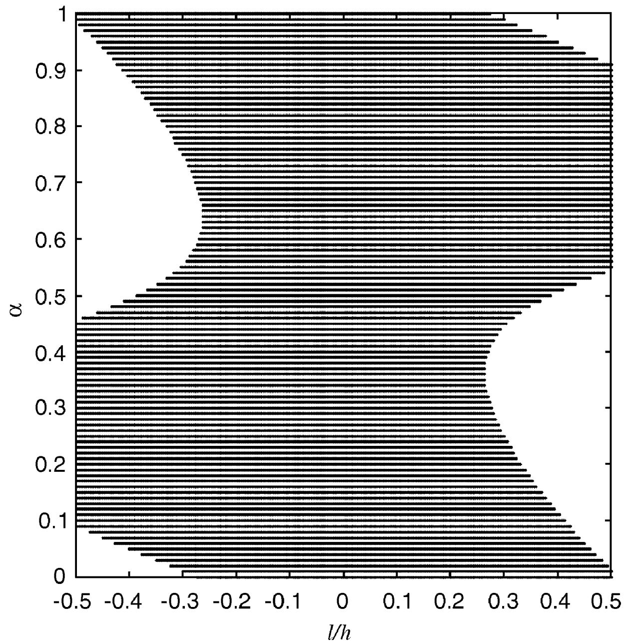

Fig. 5. Stable region (black) for stator displacement u_{Stator} of 3% of the air-gap. Stable region as function of generator position α and generator rotor offset l/h.

8.2. Rotor response

The rotor response has been analysed for rotor hub positions corresponding to $\alpha = 0.5$, 0.6 and 0.7 where $\alpha = 0.5$ means that the rotor hub is at the mid-point between the two bearings and $\alpha = 0.7$ means that the

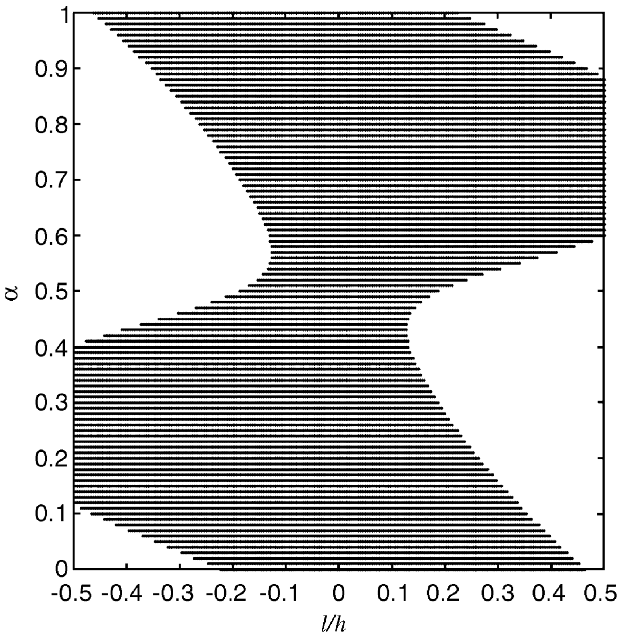

Fig. 6. Stable region (black) for stator displacement u_{Stator} of 5% of the air-gap. Stable region as function of generator position α and generator rotor offset l/h.

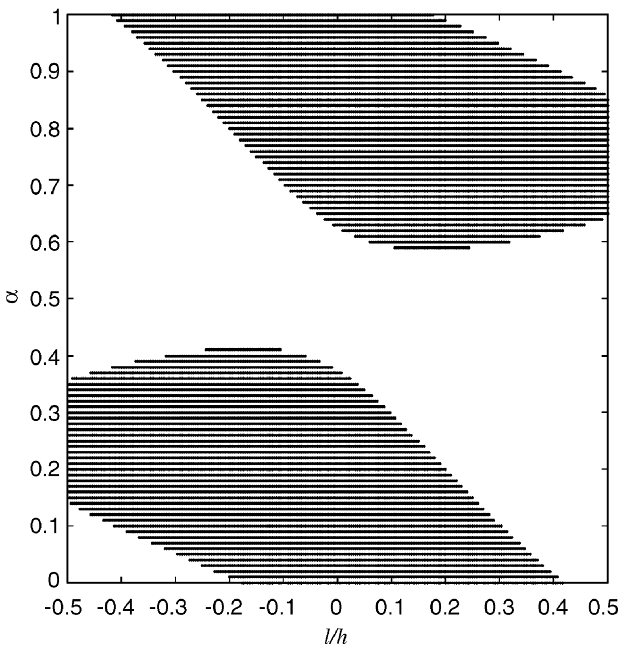

Fig. 7. Stable region (black) for stator displacement u_{Stator} of 7% of the air-gap. Stable region as function of generator position α and generator rotor offset l/h.

spider hub is closer to the lower generator bearing. The position of the spider hub on the shaft depends on the selected design of the rotor spider. In the rotor response analysis a higher value of α means that the spider hub is closer to the lower generator bearing which is common if the thrust bearing is located under the generator rotor. The ratio $l/h = 0$ means that the rotor rim centre is in the centre of the rotor spider hub. To adjust the rotor rim relative to the stator core the rotor rim may have to be moved from the centre of the spider hub. In the analysis of rotor response the rotor rim has been analysed for ratios ranging from $l/h = 0$, centre of rotor rim in centre of rotor spider hub, to $l/h = -0.4$ lifted rotor rim. To compensate for a spider hub in lower position the rotor rim centre has been lifted to keep the position of the rotor in between the bearings. In Figs. 8–10 the rotor response as well as the stator eccentricity have been scaled with the nominal air-gap length

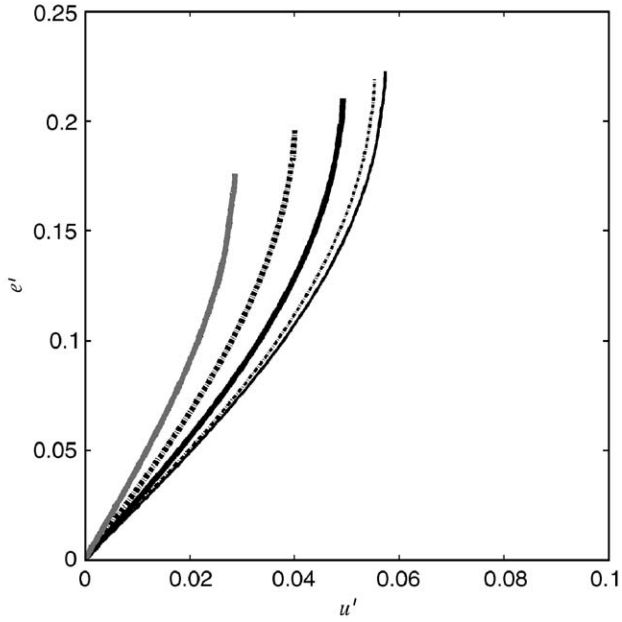

Fig. 8. Rotor response e' as function of stator eccentricity u' for generator rotor position on the shaft $\alpha = 0.5$ and for the rotor rim offset—— $l/h = 0$, …… $l/h = -0.1$, ▬▬ $l/h = -0.2$, ▪▪▪▪▪ $l/h = -0.3$, ▬▬ $l/h = -0.4$.

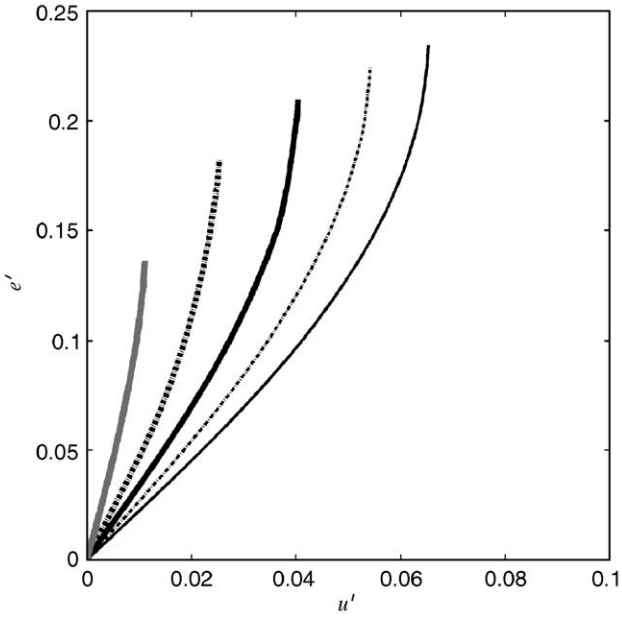

Fig. 9. Rotor response e' as function of stator eccentricity u' for generator rotor position on the shaft $\alpha = 0.6$ and for the rotor rim offset—— $l/h = 0$, …… $l/h = -0.1$, ▬▬ $l/h = -0.2$, ▪▪▪▪▪ $l/h = -0.3$, ▬▬ $l/h = -0.4$.

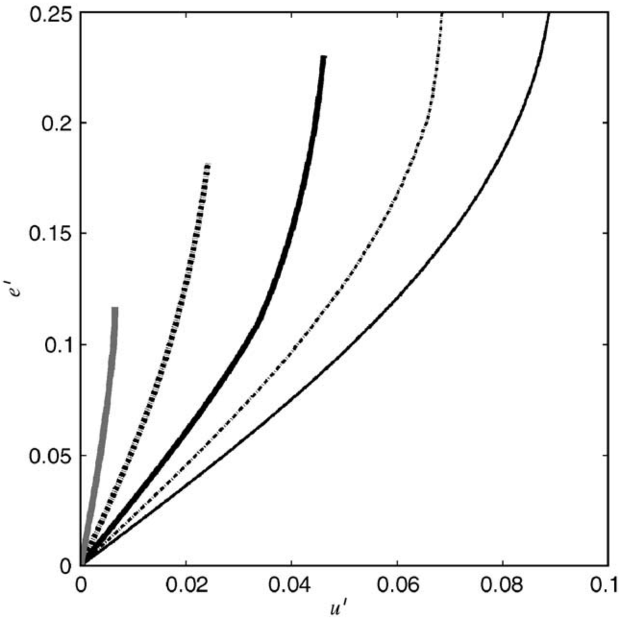

Fig. 10. Rotor response e' as function of stator eccentricity u' for generator rotor position on the shaft $\alpha = 0.7$ and for the rotor rim offset—— $l/h = 0$, …… $l/h = -0.1$, ▬▬ $l/h = -0.2$, ▪▪▪▪▪▪ $l/h = -0.3$, ▬▬ $l/h = -0.4$.

Δr, $e' = \sqrt{u_1^2 + u_2^2}/\Delta r$ and $u' = \sqrt{u_{ie_1}^2 + u_{ie_2}^2}/\Delta r$. In these figures, the rotor responses for three generator spider hub positions on the shaft, $\alpha = 0.5$, 0.6 and 0.7 have been plotted as a function of the stator eccentricity.

The rotor responses for five displaced generator rim positions have been calculated in each figure, for $l/h = 0.0$, -0.1, -0.2, -0.3 and –0.4.

9. Discussion

The rotor stability and rotor response of a generator rotor are normally analysed for a linear magnetic pull force applied in the centre of the rotor spider hub. The forces acting on the spider hub by a rotor where the centreline of rotor spider hub and the centreline of rotor rim do not coincide are normally omitted. In addition the inclination between the stator and the generator rotor is also normally omitted from analyses.

The rotor stability has been analysed for different rotor spider hub positions along the shaft and for each hub position the centreline of the rotor rim position have been varied. The stability analysis shows that the rotor can be unstable for relatively small initial stator eccentricities if the rotor rim position relative to the rotor hub is included in the analysis. In Fig. 6 the rotor stability has been plotted for a stator eccentricity of 5% of the air-gap. This figure shows that a rotor with the rotor hub at the mid-point between the bearings should be unstable if the rotor rim centreline deviates more than 15% from the centreline of the rotor hub. Using an analysis that does not consider the deviation of the rotor rim from the centre of the rotor hub, the result has shown that the rotor should be stable.

The rotor response in Figs. 8–10 has been calculated as function of stator eccentricity for three different generator hub positions. In each figure, five response curves have been plotted for different distances between centreline of generator spider hub and centreline of generator rim. The figures show that the rotor response will be underestimated if the deviation of the rotor rim from the centre of the rotor hub is not included in the analysis. For the mid span rotor in Fig. 8, with a deviation ratio of $l/h = -0.2$, the rotor response will be underestimated by 14% if the deviation between the rotor rim and rotor hub is not included in the analysis. Fig. 10 illustrates the corresponding value for a rotor with rotor hub position of $\alpha = 0.7$ on the shaft and it can be seen from Fig. 10 that the rotor response will be underestimated by 72% if the axial displacement of the rotor rim to the rotor hub is omitted in the analysis. Figs. 8–10 show that there are only few rotor configurations that are stable with an initial stator eccentricity that exceeds more than 5% of the air-gap if the deviation of the rotor rim from the centre of the rotor hub is included in the analysis.

The analyses presented in this paper are based on simplified rotor geometries and bearing configurations. One of the more important simplifications in the analyses is the assumption that the tilting-pad bearings are isotropic, have proportional damping and are load independent. The bearing type and load dependency have to be considered in the analysis when it can have significant influence of the result of the rotor response and rotor stability. However, the behaviour and influence from the nonlinear magnetic pull on the rotor configurations can be applied to several hydropower applications.

10. Conclusions

In this paper a model is proposed for analysis of an eccentric generator rotor subjected to a radial magnetic pull force. The radial force as well as the bending torque, both of which originate from the magnetic pull force affecting the generator shaft, are considered for the case in which the centre of the generator spider hub deviates from the centre of the generator rim. The electromechanical forces acting on the rotor appear as a negative stiffness vector, which affects the complex eigenvalues as well as the rotor response. Although a number of simplifying assumptions about the rotor geometry and bearing properties are made some important conclusion can be drawn from the analysis of the rotor stability and rotor response.

- The analysis of rotor stability shows that the generator rotor can be unstable for relatively small initial stator eccentricities when the rotor rim position relative to the rotor hub is included in the analysis. Analyses that omit the geometry of the generator rotor tend to overestimate the rotor stability.
- Analyses of rotor response show that the rotor response can be underestimated if the distance between the centre of the generator spider hub and the centre of the generator rim is omitted from the analysis.

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